

فرادرس

فرادرس از یک کلاس درس  
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## تحلیل پوششی داده ها یا DEA به همراه پیاده سازی عملی در متلب

مدرس:

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دکترای تخصصی مهندسی برق-کنترل

# Data Envelopment Analysis (DEA)

تحليل ٲوسٲسی راه

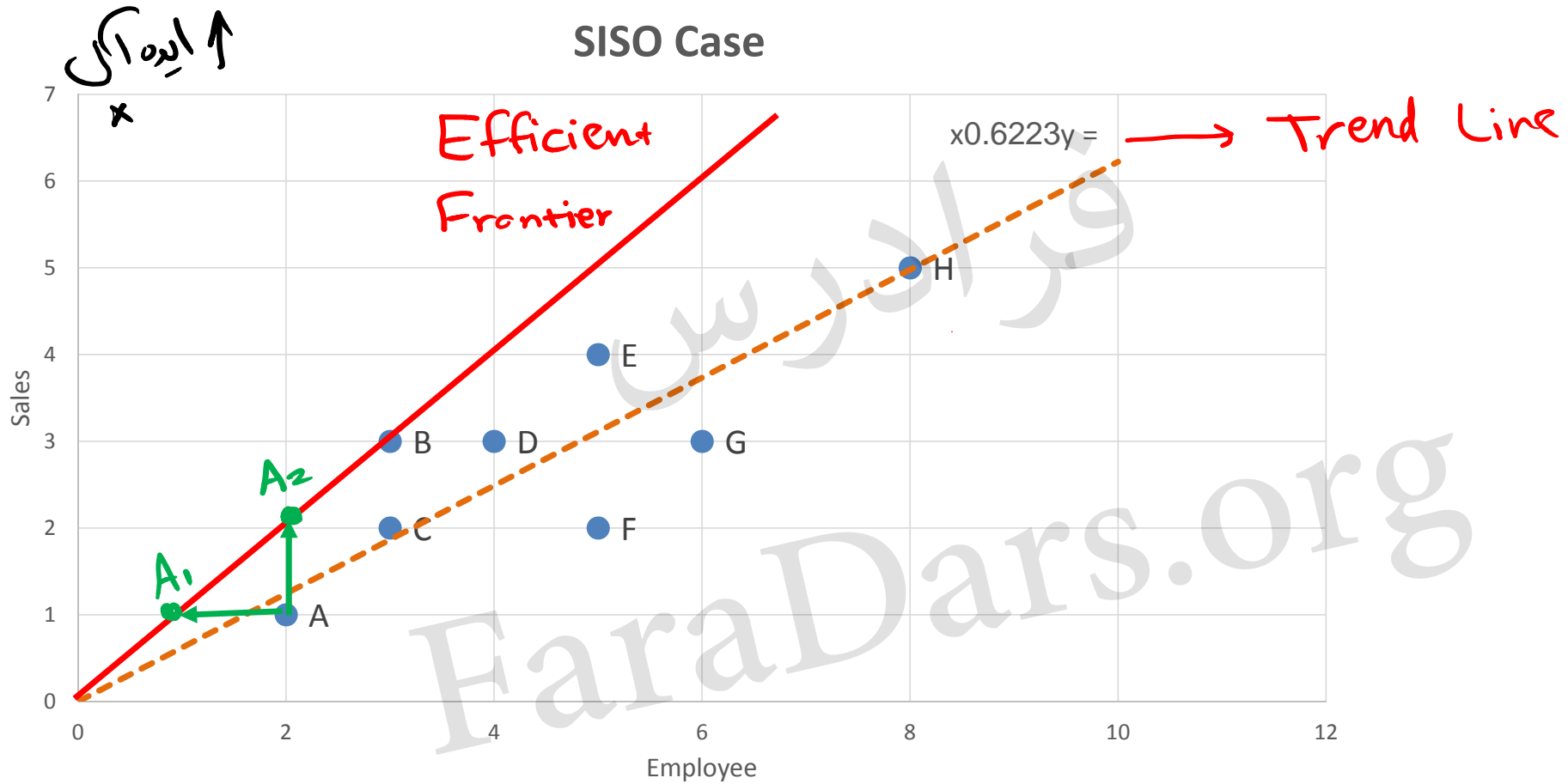
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# Decision Making Unit (DMU)



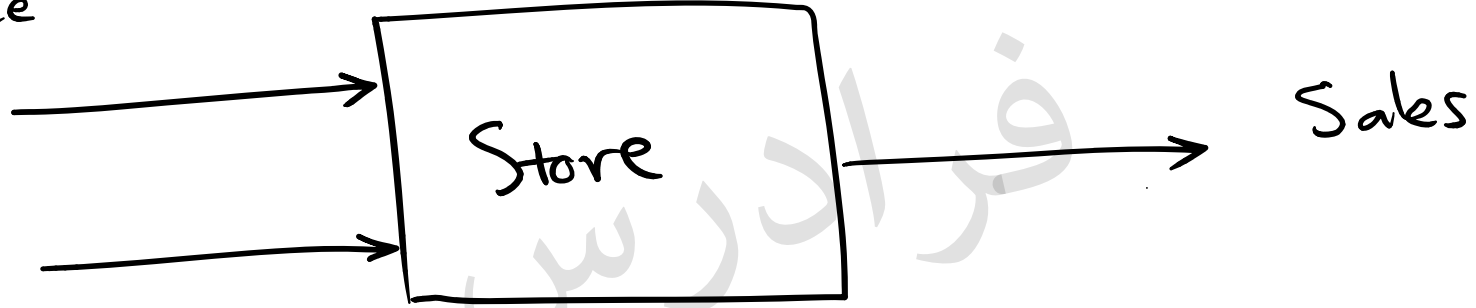
$$\text{Productivity} = \frac{\text{Output}}{\text{Input}} = \frac{y}{x}$$

# SISO Case



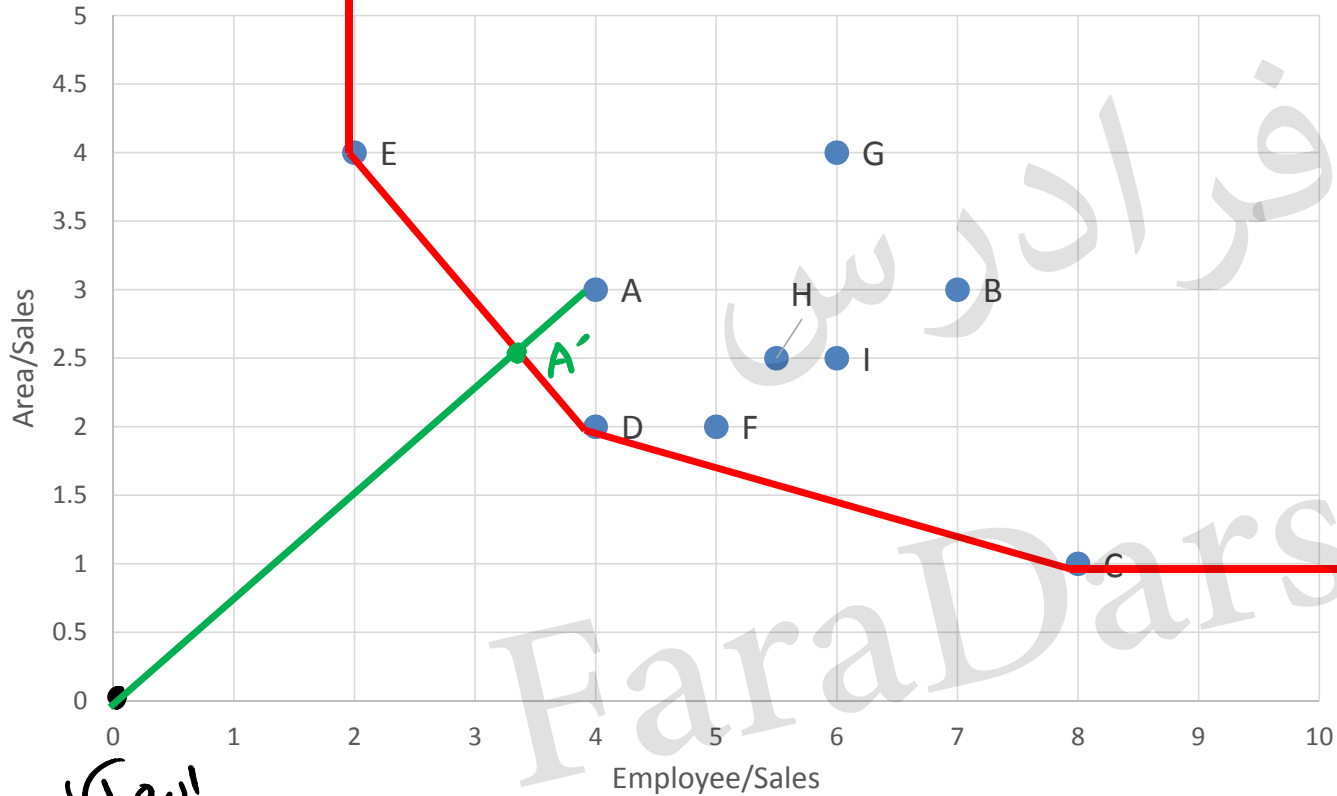
Employee

Area



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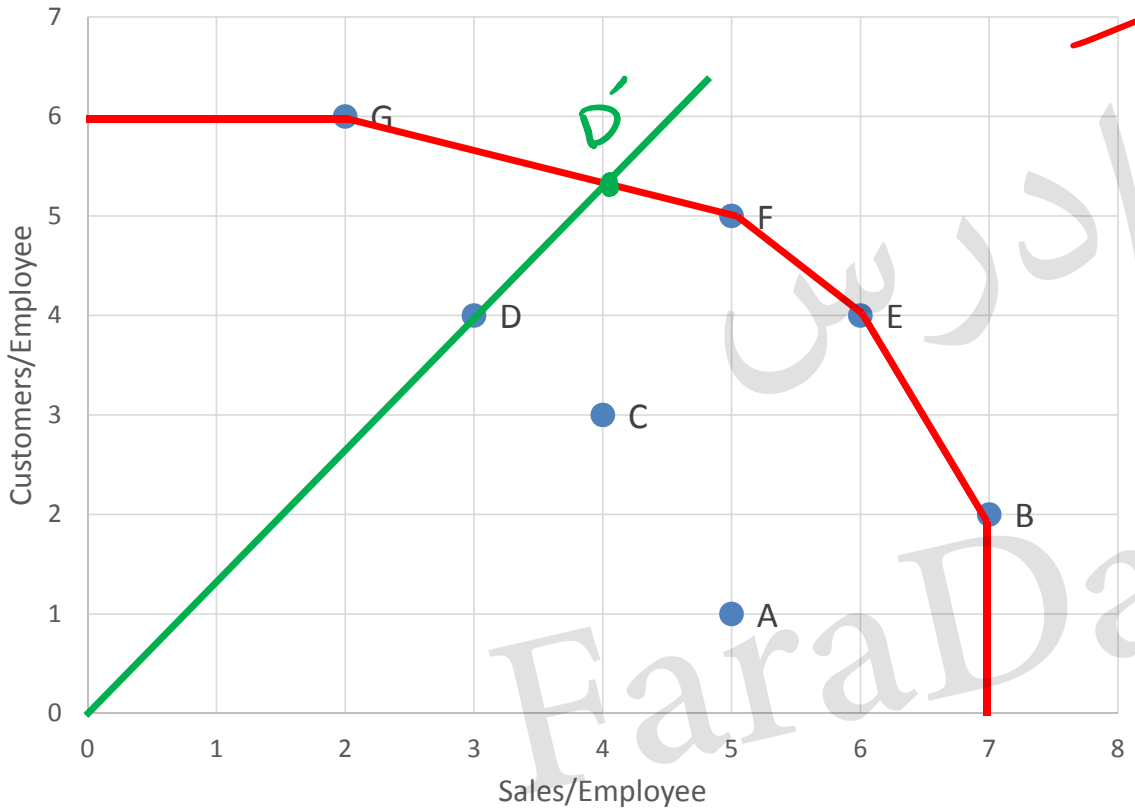
## 2 Inputs, 1 Output



$$Eff_A = \frac{\overline{OA'}}{\overline{OA}} < 1$$

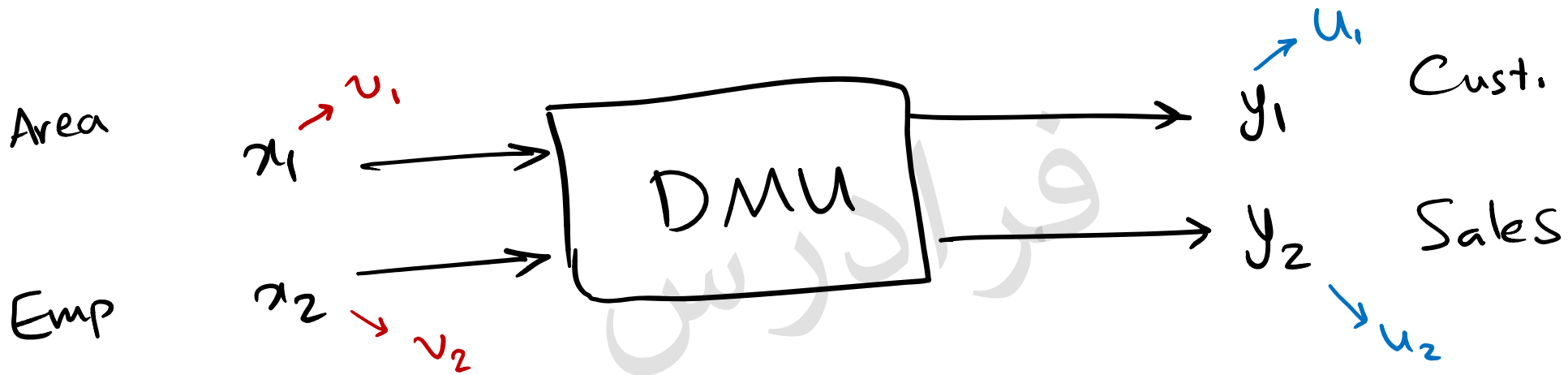
اینه

### 1 Inputs, 2 Outputs



$$Eff_D = \frac{\overline{OD}}{\overline{OD'}}$$

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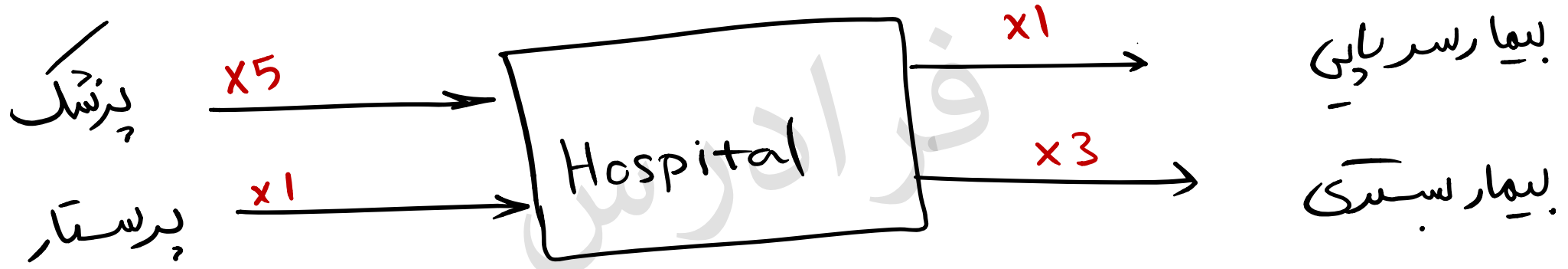


$$x = v_1 x_1 + v_2 x_2$$

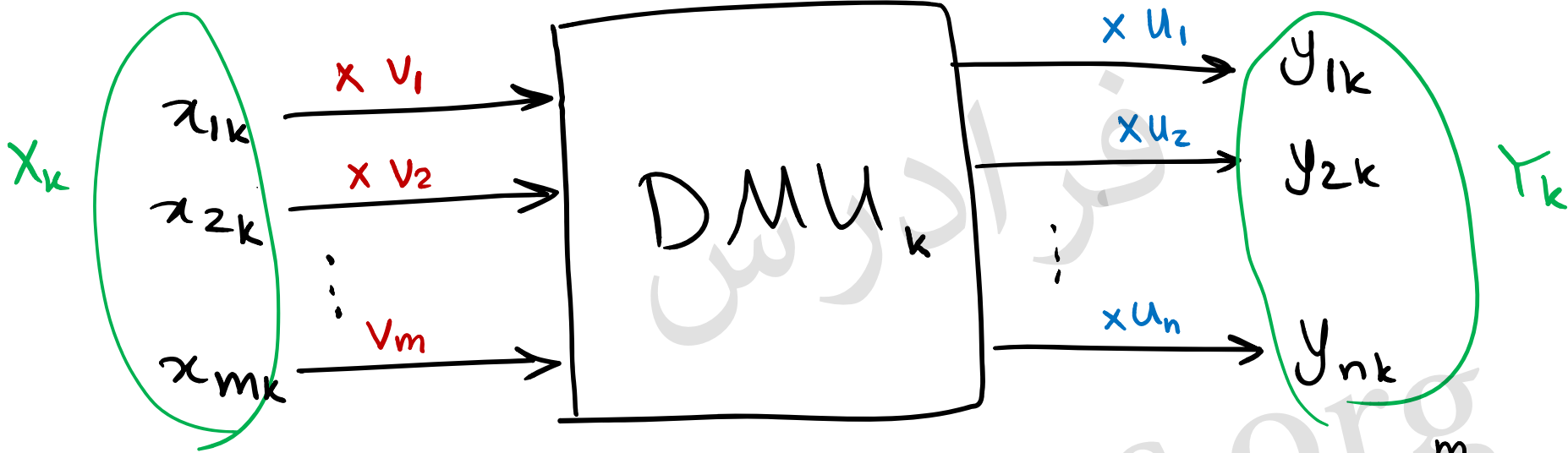
$$y = u_1 y_1 + u_2 y_2$$

$$P = \frac{y}{x} = \frac{u_1 y_1 + u_2 y_2}{v_1 x_1 + v_2 x_2}$$





$$P = \frac{1 \times \text{Outpatients} + 3 \times \text{Inpatients}}{5 \times \text{Doctors} + 1 \times \text{Nurses}}$$



Virtual Input  $X_k = v_1 x_{1k} + v_2 x_{2k} + \dots + v_m x_{mk} = \sum_{j=1}^m v_j x_{jk}$

Virtual Output  $Y_k = u_1 y_{1k} + u_2 y_{2k} + \dots + u_n y_{nk} = \sum_{i=1}^n u_i y_{ik}$

$$\text{Productivity} = \frac{\text{Virtual Output}}{\text{Virtual Input}} = \frac{Y_k}{X_k} = r_k$$

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$$\max_{u_i, v_j} r_k = \frac{Y_k}{X_k} = \frac{\sum_i u_i y_{ik}}{\sum_j v_j x_{jk}} \quad \uparrow \quad \text{CCR}$$

Subject to:

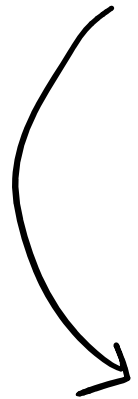
$$r_l = \frac{\sum_i u_i y_{il}}{\sum_j v_j x_{jl}} \leq 1 \quad l = 1, 2, \dots, K$$

$$u_i \geq 0 \quad i = 1, 2, \dots, n$$

$$v_j \geq 0 \quad j = 1, 2, \dots, m$$

DMU<sub>k</sub>

CCR Model for DEA



Charnes, Cooper & Rhodes (1978)

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# CCR LP Model

$$\max \quad Y_k = \sum_i u_i y_{ik}$$



DMU<sub>k</sub>

s.t.

$$X_k = \sum_j v_j x_{jk} = 1$$

$$\sum_i u_i y_{il} \leq \sum_j v_j x_{jl} \quad l=1, 2, \dots, K$$

$$u_i \geq 0 \quad \forall i \quad v_j \geq 0 \quad \forall j$$

Vectorized Form

$$\max u^T y_k$$

S.t.

$$v^T x_k = 1$$

$$u^T y_\ell \leq v^T x_\ell$$

$\rightsquigarrow$

$$v^T x_\ell - u^T y_\ell \geq 0$$

$\forall \ell$

$$u \geq 0$$

$$v \geq 0$$

$$\max \quad y_{1x} u_1 + y_{2x} u_2 + \dots + y_{nx} u_n + \underbrace{0 v_1 + 0 v_2 + \dots + 0 v_m}_{=0}$$

$$\textcircled{\min} \quad f_1 x_1 + f_2 x_2 + \dots + f_s x_s$$

$$\left. \begin{aligned} x_1 \sim x_n &\equiv u_1 \sim u_n \\ x_{n+1} \sim x_{n+m} &\equiv v_1 \sim v_m \end{aligned} \right\} n+m \text{ دوسو}$$



$$\begin{cases} x_i \leftrightarrow u_i \\ f_i \leftrightarrow -y_{ik} \end{cases} \quad i = 1, 2, \dots, n$$

$$\begin{cases} x_{n+j} \leftrightarrow v_j \\ f_{n+j} \leftrightarrow 0 \end{cases} \quad j = 1, 2, \dots, m$$

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$$A_{eq} x = b_{eq}$$

$$\underbrace{\begin{bmatrix} 0 & 0 & x_{1k} & x_{2k} \end{bmatrix}}_{A_{eq}} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{matrix} | \\ \downarrow \\ b_{eq} \end{matrix}$$

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$$Ax \leq b$$

$$\sum_i y_{il} u_i \leq \sum_j x_{jl} v_j$$

$$\sum_i y_{il} u_i - \sum_j x_{jl} v_j \leq 0 \quad l=1, 2, \dots, K$$

$$\begin{bmatrix} y_{1l} & y_{2l} & \dots & y_{nl} & -x_{1l} & -x_{2l} & \dots & -x_{ml} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_n \\ v_1 \\ \vdots \\ v_m \end{bmatrix} \leq 0$$

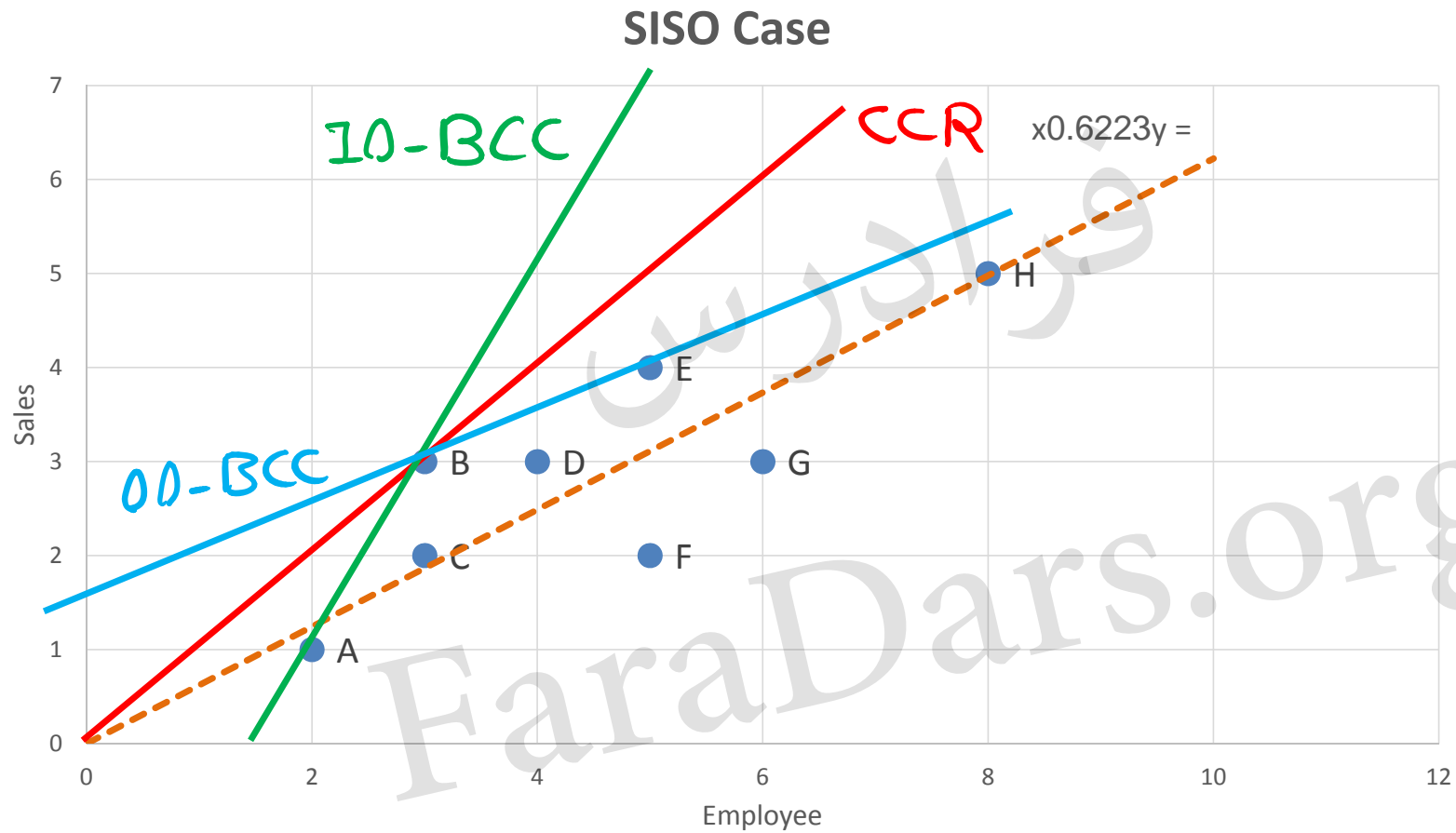
$$\begin{bmatrix}
 y_{11} & y_{21} & \dots & y_{n1} & -\lambda_{11} & -\lambda_{21} & \dots & -\lambda_{m1} \\
 y_{12} & y_{22} & \dots & y_{n2} & -\lambda_{12} & -\lambda_{22} & \dots & -\lambda_{m2} \\
 \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
 y_{1k} & y_{2k} & & y_{nk} & -\lambda_{1k} & -\lambda_{2k} & \dots & -\lambda_{mk}
 \end{bmatrix}$$

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# BCC Model

Banker, Charnes & Cooper

Input-oriented BCC Model  
Output-oriented BCC Model



max

$$\frac{\sum_i u_i y_{ik} + u_0}{\sum_j v_j x_{jk}}$$

Input-oriented BCC Model

s.t.

$$\frac{\sum_i u_i y_{il} + u_0}{\sum_j v_j x_{jl}} \leq 1 \quad \forall l$$

$$u_i \geq 0 \quad v_j \geq 0 \quad u_0 \text{ : free}$$

## IO-BCC Model

$$\max \sum_i u_i y_{ik} + u_0$$

$$\text{s.t.} \quad \sum_j v_j x_{jk} = 1$$

$$\sum_i u_i y_{il} + u_0 \leq \sum_j v_j x_{jl} \quad \forall l$$

$$u_i \geq 0 \quad v_j \geq 0 \quad u_0: \text{free}$$



max

$$\frac{\sum_i u_i y_{ik}}{\sum_j v_j x_{jk} + v_0}$$

output-oriented BCC Model

s.t.

$$\frac{\sum_i u_i y_{il}}{\sum_j v_j x_{jl} + v_0} \leq 1 \quad \forall l$$

$$u_i \geq 0 \quad v_j \geq 0 \quad v_0 : \text{free}$$

## DA-BCC Model

$$\min \sum_j v_j x_{jk} + v_0$$

$$\text{s.t.} \quad \sum_i u_i y_{ik} = 1$$

$$\sum_i u_i y_{il} \leq \sum_j v_j x_{jl} + v_0 \quad \forall l$$

$$u_i \geq 0 \quad v_j \geq 0 \quad v_0 : \text{free}$$

# Additive Model

$$\max \sum_i u_i y_{ik} - \sum_j v_j x_{jk} - v_0$$

$$\text{s.t.} \quad \sum_i u_i y_{il} \leq \sum_j v_j x_{jl} + v_0 \quad \forall l$$

$$u_i \geq 1 \quad v_j \geq 1 \quad v_0 : \text{free}$$

این اسلاید ها بر مبنای نکات مطرح شده در فرادرس  
«تحلیل پوششی داده ها یا DEA به همراه پیاده سازی عملی در متلب»  
تهیه شده است.

برای کسب اطلاعات بیشتر در مورد این آموزش به لینک زیر مراجعه نمایید.

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